3-6 Chain Rule

Learning Objectives:

I can use the chain rule to calculate derivatives.

I can use chain rule in conjunction with product and quotient rule choosing the appropriate order to calculate the derivatives.

Composite Functions

$h(x) = (3x + 1)^5$

$g(x) = 3x + 1$

$f(x) = x^5$

A composite function is a function that is made up of 2 or more functions "nessled" inside of each other.

$3x+1$ is "nessled" inside of x^5

$h(x)=f(g(x))$

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Chain Rule

When taking the derivatives of a function which is made up of the composite of 2 or more functions, take the derivative of the "outermost" function and work in.

$$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f\,'\left(g(x)\right)\cdot g\,'(x)$$

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Ex1. Find the derivative

$f(g(x))$

1.) $y = \sin(3x+5)$

$f = \sin x$ $\qquad g = 3x+5$

$f' = \cos x$ $\qquad g' = 3$

$f'(g(x)) \cdot g'(x)$ $\qquad \cos(3x+5) \cdot 3$

$\boxed{3\cos(3x+5)}$

2.) $f(x) = (5x+7)^8$

$f' = 8(5x+7)^7 \cdot 5$

$= 40(5x+7)^7$

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3.) $f(x) = \tan(x^2 + 1)$

$f' = \sec^2 x \quad g' = 2x$

$\sec^2(x^2+1) \cdot 2x$

4.) $g(x) = \cos^2(4x-6)$

$g = [\cos(4x-6)]^2$

$g' = 2[\cos(4x-6)] \cdot -\sin(4x-6) \cdot 4$

$-8\cos(4x-6) \cdot \sin(4x-6)$

5.) ~~$y = \sqrt{\tan x}$~~ $y = \sqrt{\tan(x^2-1)}$

$$y = \left[\tan(x^2-1)\right]^{1/2}$$

$$\frac{1}{\cancel{2}}\left[\tan(x^2-1)\right]^{-\frac{1}{2}} \cdot \sec^2(x^2-1) \cdot \cancel{2}x$$

$$x\left[\tan(x^2-1)\right]^{-1/2} \cdot \sec^2(x^2-1)$$

$$\frac{x\sec^2(x^2-1)}{\sqrt{\tan(x^2-1)}}$$

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6.) $y = x^2 \cos(4x^3 - 5x)$

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$y = x^2 \cos(4x^3 - 5x)$

$f = x^2$ $\quad g = \cos(4x^3 - 5x)$

$f' = 2x$ $\quad g' = -\sin(4x^3 - 5x) \cdot (12x^2 - 5)$

$$\boxed{y' = 2x\cos(4x^3 - 5x) - x^2[\sin(4x^3 - 5x)](12x^2 - 5)}$$

7.) $y = \sin(x^2 \sec x)$

main chain (product)

$y = \sin(x^2 \sec x)$

$y' = \cos(x^2 \sec x) \cdot [(2x \cdot \sec x) + (x^2 \cdot \sec x \tan x)]$

$f = x^2$ $\quad g = \sec x$

$f' = 2x$ $\quad g' = \sec x \cdot \tan x$

8.) $y = \dfrac{x^3}{\sin(x^2)}$

8.

$y = \dfrac{x^3}{\sin(x^2)}$ $\quad f = x^3$ $\quad g = \sin(x^2)$

$f' = 3x^2$ $\quad g' = \cos(x^2) \cdot 2x$

$(x^3 \cdot \cos(x^2) \cdot 2x)$

$$y' = \frac{(3x^2 \sin(x^2) - 2x^4 \cos(x^2))}{\sin^2(x^2)}$$

Ex2. Find the equation of the line tangent to y = $\cos^4 x$ at $x = \frac{\pi}{6}$

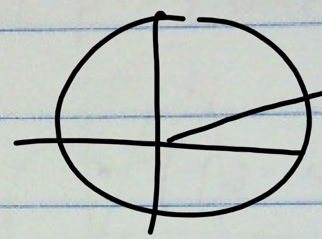
$(\sin x^2)^2$

$y = (\cos x)^4$ at $x = \frac{\pi}{6}$

$y' = 4(\cos x)^3 \cdot -\sin(x)$

$m = 4\left(\frac{\sqrt{3}}{2}\right)^3 \cdot -\frac{1}{2}$

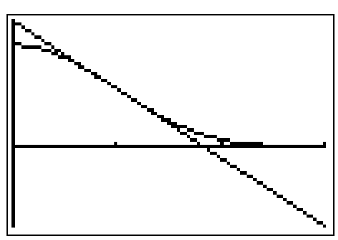
$m = -2\left(\frac{\sqrt{3}}{2}\right)^3$ $\quad \frac{2 \cdot 3\sqrt{3}}{8} = -\frac{3\sqrt{3}}{4}$



$y - \frac{9}{16} = -\frac{3\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)$

$y - \left(\frac{\sqrt{3}}{2}\right)^4 = -2\left(\frac{\sqrt{3}}{2}\right)^3\left(x - \frac{\pi}{6}\right)$

Plot1 Plot2 Plot3
\Y1 = $(\cos(X))^4$
\Y2 = $-3\sqrt{3}/4*(X-\pi/$
\Y3 =
\Y4 =



Homework

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